

The Euler, Lagrange, and Kovalevskaya Tops

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A classical problem in mechanics is the motion of a rigid body, such as a spinning top, about a fixed point under the influence of gravity. In general, this system is not integrable and may exhibit chaotic behavior. However, several remarkable integrable cases are known, including the Euler, Lagrange, and Kovalevskaya tops.

The easy case is the Euler top; this case describes a free top without any particular symmetry moving without external torque and for which the fixed point is the center of gravity. The Lagrange top describes a rigid body with rotational symmetry about an axis passing through the fixed point. Two principal moments of inertia are equal, and the center of mass lies on the symmetry axis. The Kovalevskaya top is a more challenging integrable case. It describes a rigid body whose principal moments of inertia satisfy

$$I_1 = I_2 = 2I_3.$$

In addition, the center of mass lies in the plane perpendicular to the symmetry axis and passing through the fixed point.

These examples also provide a natural introduction to precession and nutation, two important phenomena in rigid-body dynamics and astronomy. For an astronomical explanation, see the accompanying [video](#).

The goal of this project is to introduce students to rigid-body dynamics and the theory of integrable systems through these three classical examples.